Proceedings of the 20th Inverse Problems Symposium
East Lansing, Michigan, USA
11-12 June 2007

FILTER CONCEPTS AND INSIGHTS FROM FILTER COEFFICIENTS

James V. Beck
Department of Mechanical Engineering
Michigan State University
East Lansing, Michigan 48824, USA
Email: beck@msu.edu

INTRODUCTION

This paper is concerned with the efficient solution of the inverse heat conduction problem (IHCP) which is the determination of the surface heat flux from transient temperature measurements inside a solid. Repetitive or continuous measurements of heat flux can occur in both manufacturing and experiments. In such cases the filter method given in this paper is very effective and relatively simple to implement after the coefficients have been obtained. The "expert" aspects of determining the coefficients can be separated from the actual implementation on a device.

The method is based on the principle of linear superposition. It has been demonstrated that many problems with temperature-varying thermal properties, which are inherently non-linear, can be approximated satisfactorily using the concept of local linearization [1]. In the IHCP many sources of errors are possible including inaccurate location of the sensors, imperfect knowledge of the thermal properties, measurement errors caused by the sensor itself and biases introduced by the regularization of the ill-posed aspects of the IHCP. These uncertainties may be larger than errors introduced by using local linearization.

ANALYSIS

Consider a 1D heat conducting body which is heated on just one surface with a time-varying heat flux q_i. The symbol i is for the i^{th} time step given by $t_i = i\Delta t$. The temperature T_M at time t_M and at a known point inside the body can be described by

$$T_M = \sum_{i=1}^{M} X_{M+1-i} q_i = X_M q_1 + \cdots + X_1 q_M \qquad (1)$$

The X_i symbols denote the temperature rise for a unit value of the heat flux for the basis function selected for the heat flux, such as a hat-type function. This equation is in the form of a convolution which has one more term as the time index M increases by one. In general, the computational load increases as the index increases. The filter equation given in the paper has a similar characteristic with an important difference. As the computations progress in time, the computation of the heat flux of the heat flux at the "present" time, t_M, is less and less affected by the "distant" past.

The solution presentation is expedited using a matrix notation for eq. (1) given by

$$\mathbf{T}=\mathbf{Xq}, \quad \mathbf{T}^T = \begin{bmatrix} T_1 & T_2 & \cdots & T_M \end{bmatrix}$$

$$\mathbf{q}=\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_M \end{bmatrix}, \quad \mathbf{X}=\begin{bmatrix} X_1 & 0 & 0 & \cdots & 0 \\ X_2 & X_1 & 0 & \cdots & 0 \\ X_3 & X_2 & X_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ X_M & X_{M-1} & X_{M-2} & \cdots & X_1 \end{bmatrix} \qquad (2)$$

Notice that $\mathbf{X}$ is a square matrix and lower triangular with the same components on a given diagonal. If the ordinary least squares method is used to estimate the heat flux components, the results are unsatisfactory as the time steps become small.

One way to reduce the variability of the estimated values is to use Tikhonov regularization by minimizing the whole-domain function

$$S = (\mathbf{y}\text{-}\mathbf{Xq})^T(\mathbf{y}\text{-}\mathbf{Xq}) + \alpha_T(\mathbf{Hq})^T\mathbf{Hq} \qquad (3)$$

where $\mathbf{y}$ is the measured temperature vector, $\mathbf{H}$ matrix is the Tikhonov regularizing matrix and α_T is the Tikhonov regularizing constant multiplier. Minimizing eq. (3) with respect to $\mathbf{q}$ gives the estimated heat flux vector of

$$\hat{\mathbf{q}} = (\mathbf{X}^T\mathbf{X} + \alpha_T\mathbf{H}^T\mathbf{H})^{-1}\mathbf{X}^T\mathbf{y} \qquad (4)$$

This equation is linear in terms of the temperature vector $\mathbf{y}$. It is also true that any component of the estimated heat flux vector is only a function of a limited number of past (m_p) and future (m_f) temperatures. The paper gives a derivation to show that heat flux at a given time can be computed by a moving- average type of summation given by

$$\hat{q}_M = \sum_{i=M+1-m_p}^{M+m_f} y_i g_{M+1+m_f-i}$$
$$= y_{M+1-m_p} g_{m_p+m_f} + \cdots + y_M g_{m_f+1} + \cdots + y_{M+m_f} g_1 \qquad (5)$$

The filter coefficients g_i are the same for each time index M. Using eq. (5) is much simpler than using eq. (4) but gives

Address all correspondence to
James V. Beck, beck@msu.edu.

almost identical values. The *g* functions are investigated and the effectof measurement errors are considered, although the effects of random errors are almost identical to those produced by using eq. (4).

The method can be extended to multiple interior temperature sensors and to multi-dimensional problems with several time-varying heat flux components

REFERENCES

[1] Beck, James, 2007, "Filter solutions for the nonlinear inverse heat conduction problem," *Inverse Problems in Science and Engineering,* to be published 2007.